Written Exam at the Department of Economics summer 2020-R

Pricing Financial Assets

Final exam

August 20, 2020

(3-hour open book exam)

Answers only in English.

The paper must be uploaded as <u>one PDF document</u>. The PDF document must be named with exam number only (e.g. '127.pdf') and uploaded to Digital Exam.

This exam question consists of 3 pages in total

This exam has been changed from a written Peter Bangsvej exam to a take-home exam with helping aids. Please read the following text carefully in order to avoid exam cheating.

Be careful not to cheat at exams!
You cheat at an exam, if you during the exam:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text. This also applies to text from old grading instructions.
- Make your exam answers available for other students to use during the exam
- Communicate with or otherwise receive help from other people
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Use parts of a paper/exam answer that you have submitted before and received a passed grade for without making use of source referencing (self plagiarism)

You can read more about the rules on exam cheating on the study information pages in KUnet and in the common part of the curriculum section 4.12.

Exam cheating is always sanctioned with a warning and dispelling from the exam. In most cases, the student is also expelled from the university for one semester.

The Exam consists of 4 problems. The first problem will enter the evaluation with a weight of 40%; the others with 20% each.

1. Consider an economy where the value of a bank account B_t at time and the price of a non-dividend paying stock S_t at time t are described by the Ito-processes:

$$dB_t = rB_t dt \text{ , } B_0 = 1$$
$$dS_t = \mu S_t dt + \sigma S_t dz \text{ , } S_0 > 0$$

   where μ i constant, σ is a positive constant, r is a constant risk free rate, and dz is the standard short hand notation for a Brownian increment.

   a) Why can we use S_t as a numeraire?
   b) Consider the process B_t/S_t and express this as an Ito process
   c) What μ will make this a martingale?
   d) What will the drift rate of lnS_t be under this numeraire?
   e) A derivative pays $S_T G(S_T)$ at time $T > t$, where G is some function. With E^S denoting the expectation under the probability measure corresponding to S_t as a numeraire find an expression for the price f_t of this derivative at time t.

2. Let $y(0, T)$ be the continuously compounded yields at time $t = 0$ on zero coupon bonds with maturities $T > 0$, i.e. with prices $P(0, T) = \mathsf{e}^{-y(0,T)T}$.

   a) Find the instantaneous forward rate $f(0, T)$ as contracted at 0 for maturity T (i.e. for a period $(T, T + dT)$).
   b) Show that if y is increasing in T, then $f(0, T) > y(0, T)$ and interpret this result.

3. Consider a one-period coupon bond that promises to pay its principal (of 1) and the accrued interest from a (continuously compounded) coupon of c at time $t = 1$. Assume that there is a risk free (continuously compounded) interest rate of r.

   Suppose that the bond will default with $\mathbb{Q}$-probability q with a known fractional recovery of $R, 0 < R < 1$ (of principal and accrued interest).

   Use a one-period binomial model to find the coupon c that will give the defaultable bond a par price at time $t = 0$.

   What is the credit spread?

4. Consider a European call option on a stock with price S_t at time t. Assume that the stock pays no dividends before the expiry of the option.

   a) Define the delta Δ of the call.
   b) For an at-the-money call (where the strike K of the option is equal to S_t) the delta is close to $1/2$. Use the Black-Scholes-Merton call option formula to find the strike K where the delta is precisely $1/2$. For your reference the call option formula can be written

$$c_t = S_t \Phi(d_1) - K\mathsf{e}^{-r(T-t)}\Phi(d_2)$$

   where

$$d_1 = \frac{\ln \frac{S_t}{K} + (r + \frac{1}{2}\sigma^2)(T - t)}{\sigma\sqrt{T - t}}$$
$$d_2 = d_1 - \sigma\sqrt{T - t}$$

where r is a constant continuously compounded interest rate, T is the expiry date of the option and σ the constant volatility. $\Phi(x)$ denotes the standard normal distribution function in x.

c) Assume $S_t = 1$ and find this strike for parameters $(T - t) = 0.25$, the volatility $\sigma = 0.4$ and the risk free interest rate $r = 0$.

d) What is the delta of the corresponding put option?